RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College affiliated to University of Calcutta)

B.Sc. FOURTH SEMESTER TAKE-HOME TEST/ASSIGNMENT, AUGUST 2021
SECOND YEAR [BATCH 2019-22]

Date : 11/08/2021 **MATHEMATICS**
Time : 11am – 1pm **Paper MACT 10** Full Marks : 50

Instructions to the Candidates

- Write your **College Roll No, Year, Subject & Paper Number** on the top of the **Answer Script**.
- Write your **Name, College Roll No, Year, Subject & Paper Number** on the **text box of your e-mail**.
- Read the instructions given at the beginning of each group/unit carefully.
- Only handwritten (by blue/black pen) answer-scripts will be permitted.
- Try to answer all the questions of a single group/unit at the same place.
- All the pages of your answer scripts must be numbered serially by hand.
- In the last page of your answer-scripts, please mention the total number of pages written so that we can verify it with that of the scanned copy of the scripts sent by you.
- For an easy scanning of the answer scripts and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the exam, scan the entire answer script by using Clear Scan: Indy Mobile App OR any other Scanner device and make a **single PDF file (Named as your College Roll No)** and send it to [redacted]

Group A : Analytical Statics

Answer Question number 1 and any three from 2 to 5 :-

1. Define angle of friction. State the relationship between the angle of friction and the coefficient of friction. [2]

2. Three forces P, Q, R act along the sides of the triangle formed by the lines

$$x + y = 3; \quad 2x + y - 1 = 0 \quad \text{and} \quad y - x = 1$$

in order. Find the line of action of their resultant. [6]

3. Four heavy equal uniform rods are freely jointed so as to form a rhombus which is freely suspended by an angular point. The midpoints of the two upper rods are connected by a light rod so that the rhombus cannot collapse. Prove by the principal of virtual work that the thrust in the light rod is $W \tan\alpha$, where W is the total weight of the four rods and 2α is the angle of the rhombus at the point of suspension. [6]

4. A uniform lamina in the form of an equilateral triangle ABC of side $\frac{4a}{\sqrt{3}}$ are in equilibrium with BC on top of a fixed sphere of radius a. What is the greatest weight which can be attached to the vertex A so that the equilibrium may remain stable? [6]

5. The ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is placed with the axis of x is vertical and its surface is rough. Show that a heavy particle will rest on it anywhere above its intersection with the cylinder

$$\frac{y^2}{b^2}\left(1 + \frac{a^2}{\mu^2 b^2}\right) + \frac{z^2}{c^2}\left(1 + \frac{a^2}{\mu^2 c^2}\right) = 1,$$

where μ is the coefficient of friction. [6]

Group B : Analytical Dynamics of a particle

Answer Question number 6 and any two from 7 to 9 :-

6. A particle moves in a straight line OCP being attracted by a force $m\mu$. PC is always directed towards C, whilst C moves along OC with a constant acceleration f; here P is the position of the particle at time t. If initially C was at rest at the origin O and the particle was at a distance b from O moving with a velocity V, then prove that the distance of the particle from O at any time t is [6]

$$\left(\frac{f}{\mu}+b\right)\cos(t\sqrt{\mu})+\frac{V}{\sqrt{\mu}}\sin(t\sqrt{\mu})-\frac{f}{\mu}+\frac{1}{2}ft^2$$

OR

A particle subject to gravity is projected at an angle α with the horizontal in a medium which produces a retardation equal to k times the velocity. It strikes the horizontal plane through the point of projection at an angle ω, and the time of flight is T. Prove that

$$\frac{\tan\omega}{\tan\alpha}=\frac{e^{kT}-1-kT}{e^{-kT}-1+kT}$$

and deduce that $\omega > \alpha$. [6]

7. (a) Establish a criterion for the stability of a circular orbit of radius c, with centre at the centre of force, described under the influence of a force $f(r)$. [6]

   (b) A particle is projected vertically upwards with a velocity $\frac{g}{k}$ in a medium whose resistance is kv per unit mass , where v is the velocity of the particle and g is the acceleration due to gravity. Find the height attained by the particle after time $\frac{1}{k}$. [6]

8. (a) A smooth straight tube rotates in a horizontal plane with angular velocity ω about a fixed end, and a particle moves within it under a resistance equal to k times the square of the relative velocity. Prove that if the particle be projected so as to come to rest at the fixed end, the relative velocity at the distance r from that end is [6]

$$\frac{\omega}{k\sqrt{2}}\sqrt{e^{2kr}-2kr-1}$$

   (b) If a particle moves under attractive central acceleration $\frac{\mu}{r^5}$, then prove that the velocities v_1, v_2 at the two apsidal distances satisfy the relation $v_1^2+v_2^2=\frac{2h^4}{\mu}$, where h is the constant angular momentum about the centre of force for a particle of unit mass. [6]

9. (a) If V_1 and V_2 are the velocities of a planet when it is, respectively, nearest and farthest from the Sun, prove that $(1-e)V_1=(1+e)V_2$, where e is the eccentricity of the planet's orbit. Also, prove that if V be the velocity at the end of the minor axis of the planet's orbit then V is the geometric mean of V_1 and V_2. [4+2]

   (b) A particle describes an elliptic path under an acceleration always directed towards the minor axis of the ellipse. Find the law of force. [6]

—— X ——